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DEPARTMENT OF ECONOMICS  
RESEARCH MEMORANDUM

**EQUILIBRIUM WITH CO-ORDINATION AND  
EXCHANGE INSTITUTIONS: A COMMENT**

W. Spanjers

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# Equilibrium with Co-ordination and Exchange Institutions: A Comment<sup>1</sup>

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### Abstract

In this comment an economy with exchange institutions is defined as a special kind of a social system with co-ordination. In such an economy consumers co-ordinate their trades through the exchange institutions.

It is shown that the theorems of Vind (1983) and Keiding (1985) on the existence of an equilibrium in a social system with co-ordination have to be amended. The amended version of these theorems is used to derive an existence theorem for an economy with exchange institutions.

It is stated that, under certain conditions, equilibria in an economy with exchange institutions support and only support allocation that are Pareto efficient and individually rational for the consumers.

This result is used to find out whether the following allocation mechanisms viz. the core, the Walrasian market and the monopoly market can be supported by exchange institutions. We find that this is not the case for the monopoly market only.

# 1 Introduction

Social systems were introduced by Debreu (1952). Arrow and Debreu (1954) proved the existence of equilibrium in a competitive economy by interpreting it as a social system, or an abstract economy, and then applying an equilibrium existence theorem for abstract economies.

Vind (1983) added co-ordination to the social system, thus extending the possible use of abstract economies to economic models in which co-ordination plays an important rôle. Co-ordination is a natural phenomenon in models in which external effects occur. An existence theorem for equilibrium in a social system with co-ordination was provided. The social system with co-ordination was used to prove the existence of an equilibrium in an economy with bilateral exchanges. In this economy all trade is effected through bilateral exchange institutions which are co-ordinated by the consumers that are allowed to use them. As Vind noticed this application has some drawbacks. Some of the equilibria fail to be sustainable under voluntary exchange.

Vind (1986) pointed out that proving existence of equilibrium in an economy with bilateral exchanges presents some difficulties if one requires all exchanges, that is all the use of the exchange institutions, to be voluntary. In the case of voluntary exchanges it is assumed that no consumer wants to drop (a fraction of) the exchanges effected through an exchange institution. It was shown that Walrasian equilibria correspond to voluntary exchange equilibria in a bilateral exchange economy with money.

The paper of Grodal and Vind (1989) gives an application of a social system with co-ordination to the field of missing markets. Pre-markets are introduced as institutions through which exchanges of certain goods can take place. Furthermore some agents may be able to use a premarket for their exchanges whereas others may not. A market is defined to be a premarket with a given price vector. Trades through a market are assumed to be possible only if for every agent using the market the net trade vector has value zero. This leads to an economy which consists of a set of consumers and a set of markets.

In this comment we address two questions. Firstly we show that the existence theorems for equilibrium in a social system with co-ordination of Vind (1983) and Keiding (1985) have to be amended. Secondly we analyse whether an improved ver-

sion of Vind's exchange institutions supports the following allocation mechanisms, viz. the core, the Walrasian market and the monopoly market. We arrive at a negative answer for the last mechanism only.

The organization of this comment is as follows.

In Section 2 we define an economy with exchange institutions as a social system with co-ordination consisting of a set of consumers, who are described by their preference relations and their initial endowments, and a set of exchange institutions. These exchange institutions are defined in a similar way as in Vind (1983). An exchange institution enables the consumers participating in it to exchange all commodities with each other and assure that the sum of the net-trades through it equals zero. The consumers co-ordinate their exchanges through the exchange institutions. We do not require voluntary exchange, but instead assume voluntary participation in the economy with exchange institutions. Thus the notion of individual rationality in an economy with external effects is captured.

In Theorem 3.1 we amend the existence theorem of Keiding (1985, Theorem 2) for equilibrium in a social system with co-ordination. Our proof is, essentially, that of Keiding. We note that this theorem is not, contrary to Keiding's claim, an extension of the existence theorem of Vind (1983, Theorem 3). Also, we show that the latter result is incorrect. The error invalidates the proof of Vind's (1983, Theorem 5) equilibrium existence result for an economy with bilateral exchanges. Therefore, we use our Theorem 3.1 to derive an equilibrium existence theorem for an economy with exchange institutions (Theorem 3.2). Our method of proof can also be used for a correct proof of Theorem 5 of Vind (1983).

In Section 4 we show that if an economy with exchange institutions is connected and the preferences of the consumers are individualistic, then the equilibria in this economy support those and only those Pareto efficient allocations that are individually rational. We use this equivalence result to show that core allocations can be supported as equilibria in the economy that has the coalitions as its exchange institutions. We also show that Walrasian equilibria can be supported as equilibria in an economy which has the Walrasian market as its sole exchange institution. Finally we show that a monopolistic market cannot in general be supported by an exchange institution.



## 2 The Model

In the first subsection a definition of a social system with co-ordination is given and equilibrium is defined. The second subsection is used to define an economy with exchange institutions and connectedness of such an economy.

### 2.1 The Social System with Co-ordination

We define a social system with co-ordination and an equilibrium in such a system. Before giving a formal definition we discuss the notion of co-ordination.

If someone co-ordinates the actions of his subordinates, what happens? First of all he is assumed to know about the (relevant) actions of his subordinates. One might assume some course of action is planned by the co-ordinator for each of his subordinates. If some subordinate deviates from the actions the co-ordinator planned for him, he may in deviating undertake actions conflicting with the actions of other subordinates or of the co-ordinator. The combination of his new actions and the actions of the others might lead to a less preferred situation for the co-ordinator.

The crucial aspect of co-ordination is that the subordinate is only allowed to undertake actions different from the planned actions if his co-ordinator agrees to the change. The co-ordinator, the boss, can be said to have the right to veto any deviations from the planned actions he co-ordinates. An agent may have more than one co-ordinator. If a change in the system of actions is proposed every agent takes into account the changes in his own actions and the changes in the actions of the agents he co-ordinates.

An equilibrium system of actions is required to be feasible and stable. Stability is defined to mean that no change in actions can be proposed that leads to anticipated states that are attainable and not anticipated to be vetoed by any agent.

This leads to the following formalization.

**Definition 2.1** *A Social System with Co-ordination is an indexed family*

$\Gamma := (X, \{\beta_a, P_a, e_a\}_{a \in A})$  *where:*

1. *A is a finite non-empty set.*
2. *X is a nonempty set.*

3.  $\beta_a : X \rightrightarrows X$  is a correspondence. We define  $\beta : X \rightrightarrows X$  by  $\beta(x) = \bigcap_{a \in A} \beta_a(x)$ .
4.  $P_a : X \rightrightarrows X$  is a correspondence such that  $x \notin P_a(x)$ .
5.  $e_a : X \times X \rightarrow \mathbb{R}^I$  is a function.

We give the social system with co-ordination,  $\Gamma$ , the following interpretation. The set  $A$  is the set of agents. The set  $X$  is the set of actions or states available to the social system as a whole. As an example we might define the set of states of the system as  $X := \prod_{a \in A} X_a$ , where  $X_a$  is the set of actions available to agent  $a$ . The correspondence  $\beta_a$  assigns to each  $x \in X$  the set  $\beta_a(x)$  of states attainable for agent  $a$  starting from state  $x$ . The correspondence  $P_a$  assigns to each state  $x \in X$  the set  $P_a(x)$  of states strictly preferred to  $x$  by agent  $a$ . For the pair of states  $(x, y) \in X \times X$  we interpret  $e_a(x, y)$  as the state anticipated by agent  $a$  to be obtained when instead of state  $x$  state  $y$  is proposed.

Now that we have defined a social system with co-ordination, we define an equilibrium in it. We define  $M : X \times X \rightrightarrows A$  such that  $M(x, y) = \{a \in A \mid e_a(x, y) \neq x\}$  and define  $I : X \rightrightarrows X$  such that  $I(x) = \{y \in X \mid M(x, y) \neq \emptyset\}$ . We interpret  $M(x, y)$  as the set of agents who are informed about a planned change in state from  $x$  to  $y$ . The set  $I(x)$  is interpreted as the set of states  $y$  such that at least one agent is informed about a change from state  $x$  to state  $y$ . This means that a change from state  $x$  to some state  $y \in I(x)$  is not unnoticed.

**Definition 2.2** *A state  $x^* \in X$  is an equilibrium in the social system with co-ordination  $\Gamma$  if:*

1.  $x^* \in \beta(x^*)$ .
2.  $\nexists y \in I(x^*) : \forall a \in M(x^*, y) [e_a(x^*, y) \in P_a(x^*) \cap \beta_a(x^*)]$ .

Thus a state  $x^*$  is an equilibrium if and only if:

1. it is attainable given the actions  $x^*$ .
2. there does not exist a state,  $y$ , such that the change from state  $x^*$  to state  $y$  which is not unnoticed, and for every agent  $a \in M(x^*, y)$  who is informed about the change the state  $e_a(x^*, y)$  which he anticipates to result is:

- (a) attainable from his point of view.
- (b) strictly preferred by him to the state  $x^*$ .

Equilibrium in a social system with co-ordination is a generalization of the Nash equilibrium concept. In the case of a Nash equilibrium every player looks for improvements for himself, *given* the actions of the other players. One might say that in the case of Nash equilibrium players only co-ordinate their own actions or, as Vind (1983) puts it, the Nash equilibrium arises in the case of no co-ordination in the social system.

## 2.2 The Economy with Exchange Institutions

In Walras (1874) attention is paid to co-ordination, to institutions and the relation between the two of them. Walras writes<sup>1</sup>

“... for any phenomenon to be classified under the heading institutions (...) it is necessary and sufficient that this phenomenon too originate in the exercise of human will and, besides, that it consist of a relationship between persons and persons designed for the mutual co-ordination of destinies of the persons concerned.”

So Walras understands institutions to be organization forms that help to co-ordinate the actions of persons. In the context of an exchange economy one might be inclined to think of markets as institutions. Furthermore considering the core-like equilibrium concepts one might consider coalitions as institutions. We say more about this in Section 4 of this paper.

In Vind (1983) an exchange economy in which exchange takes place through bilateral exchange institutions was introduced. These exchange institutions do not have relevant preferences of their own. The consumers are assumed to co-ordinate the institutions they participate in, thus assuring these institutions seem “to be designed for the mutual co-ordination of the destinies of the persons concerned.” Therefore they seem to meet Walras’ the description.

This leads to a model of an economy with two types of agents. The first type are the *consumers*. The second type of agents are the *exchange institutions*. The exchange institutions do not really have preferences which they try to maximize.

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<sup>1</sup>Walras (1874), page 63.



Their rôle in the economy is to “supervise” the exchanges. In fact exchanges are assumed not to take place directly between consumers but to be preformed indirectly with the exchange institutions as intermediaries.

We assume the preferences of the consumers to be defined over the allocation of goods over all consumers in the economy. We could, of course, have defined the preferences such that they depend on the way in which the exchange institutions are used, that is in such a way that it matters from whom you get the commodities. This is the case if one likes to eat apples from Argentina but rejects to eat the same type of apples if they come from Chile.

In defining the set of states attainable for consumer  $c$ ,  $\beta_c$ , we require that consumer  $c$  *participates in the economy voluntarily*. In our context voluntary participation is defined as to capture the notion of individual rationality.

**Definition 2.3** *A consumer,  $c$ , with preference relation  $\succeq_c$  and initial endowments  $\omega_c \in \mathbb{R}_+^I$ , participates voluntarily in an economy  $\mathbf{E}$  which yields an allocation  $x^* \in \mathbb{R}_+^{I \times \#C}$  if and only if  $x^* \succeq_c b_c$ , where*

$$b_c \in \min_{\succeq_c} \{z \in \mathbb{R}_+^{I \times \#C} \mid z \text{ is feasible in } \mathbf{E} \text{ and } z_c = \omega_c\}$$

In assuming voluntary participation we exclude the case in which consumer  $c$  would be sure to be better off by abstaining from all trading. Note that consumer  $c$  does not take the voluntary participation of the other consumers into account in determining the worst possible outcome if he would not participate in the economy. If there are no external effects and there is only one exchange institution which consists of all consumers, voluntary participation implies voluntary exchange.

In the economy with exchange institutions we denote the set of consumers by  $C$ , whereas the set of exchange institutions is denoted by  $I$ . We denote an exchange institution by the consumers who are allowed to trade through this institution. We take as the set of institutions in our social system with co-ordination a subset of  $2^C$ , the set of all subsets of the set  $C$ . We define exchange institutions such that all participating agents are allowed to exchange all commodities through the institutions. Of course one could define exchange institutions such that agents can only exchange certain commodities through an exchange institution. One might even define them in such a way that not every agent participating in an exchange institution is allowed to exchange the same commodities through this institutions.

After suitable changes in the definitions the results of Section 3 and Section 4 still hold.

We define a co-ordination system, as described by the functions  $\{e_a\}_{a \in C \cup I}$ , such that on the one hand the exchange institutions co-ordinate the exchanges taking place through them and on the other hand consumers co-ordinate the exchanges through the exchange institutions they are part of. We use  $(x_{-i}; y_i)$  to denote the vector  $x$  with  $x_i$  replaced by  $y_i$ .

**Definition 2.4** Let  $C$  be a set of consumers with preference relations  $\succeq_c$  which are complete preorderings over  $\mathbb{R}^{l \times \#C}$ , where  $l$  denotes the number of commodities, and initial endowments  $\omega_c \in \mathbb{R}_+^l$ . Let  $I$  be a subset of  $2^C$ . Then an **Economy with Exchange Institutions**  $\mathbf{E} = (\{\succeq_c, \omega_c\}_{c \in C}, I)$  is defined to be the social system with co-ordination  $\Gamma = (Y, \{\beta_a, P_a, e_a\}_{a \in C \cup I})$ , where:

$$\text{int } Y \supset \{y : C \times I \rightarrow \mathbb{R}^l \mid \forall c \in i \in I : -\sum_{c \in C} \omega_c \leq y(c, i) \leq \sum_{c \in C} \omega_c\}.$$

For every exchange institution  $i \in I$ :

$$\beta_i(x) = \beta_i = \{y \in Y \mid \sum_{c \in i} y(c, i) = 0\}.$$

$$P_i(x) = Y \setminus \{x\}.$$

$$e_i(x, y) = (x_{-i}; y_i).$$

For every consumer  $c \in C$ :

$$\beta_c(x) = \beta_c = \{y \in Y \mid (\omega_d + \sum_{i \ni d} y(d, i))_{d \in C} \succeq_c b_c$$

$$\text{and } \sum_{i \ni c} y(c, i) \geq -\omega_c\}$$

$$\text{where } b_c \in \min_{\succeq_c} \{z \in \mathbb{R}_+^{l \times \#C} \mid \exists y \in \cap_{i \in I} \beta_i \text{ such that } \forall i \in I : y(i, c) = 0$$

$$\text{and } \forall d \in C \setminus \{c\} : z_d = \sum_{i \ni d} y(i, d) + \omega_d\}.$$

$$P_c(x) = \{y \in Y \mid \omega_c + \sum_{i \ni c} y(c, i) \succ_c \omega_c + \sum_{i \ni c} x(c, i)\}.$$

$$e_c(x, y) = (x_{i \not\ni c}; y_{i \ni c}).$$

We define equilibrium in an economy with exchange institutions to be an equilibrium in the social system with co-ordination it can be represented by. Because of the special characteristics of an economy with exchange institutions it follows that for every  $y \neq x$  it holds that  $M(x, y) := \{a \in A \mid e_a(x, y) \neq x\} \neq \emptyset$ . This leads to the following equivalent formalization of equilibrium.



**Definition 2.5** Let  $\mathbf{E} = (\{\succeq_c, \omega_c\}_{c \in C}, I)$  be an economy with exchange institutions and let  $x \in Y$ . Define

$$\mathcal{P}(x) := \{y \in Y \mid \forall a \in M(x, y) : e_a(x, y) \in P_a(x)\}.$$

$$\mathcal{B}(x) := \{y \in Y \mid \forall a \in M(x, y) : e_a(x, y) \in \beta_a\}.$$

A system of net-trades  $y^*$  is called an **equilibrium** in an economy with exchange institutions  $\mathbf{E}$  if:

1.  $y^* \in \beta(y^*)$ .
2.  $\mathcal{P}(y^*) \cap \mathcal{B}(y^*) = \emptyset$ .

The first equilibrium condition in the above formulation states that an equilibrium net-trade system must be attainable. It is a statement about the *actual* equilibrium state. The second equilibrium condition is about *anticipated* states. The set  $\mathcal{P}(x)$  is the set of net-trades that, for a given  $x$ , are anticipated to result in net-trades preferred to  $x$  by every agent that anticipates a change in net-trades. The set  $\mathcal{B}(x)$  is the set of net-trades that, for a given  $x$ , are anticipated to result in a attainable state by those agents that anticipate a change in net-trades. Condition 2 states that no net-trade exists that by every agent that anticipates a state different from  $y^*$ , is anticipated to result in a state that is both preferred to  $y^*$  and to be attainable from  $y^*$ .

In the proof of the theorems on equilibria in economies with exchange institutions we use the equivalent formulation of equilibrium in a social system with co-ordination. This formulation is less suited for the purpose of exposition but easier to work with in the proofs.

Given the exchange institutions of an economy  $\mathbf{E} = (\{\succeq_c, \omega_c\}_{c \in C}, I)$  one might wonder if it is possible for every consumer in the economy to exchange any commodity with any other consumer in the economy. If this is possible we call an economy with exchange institutions connected. Formally

**Definition 2.6** An economy with exchange institutions  $\mathbf{E} = (\{\succeq_c, \omega_c\}_{c \in C}, I)$  is said to be **connected** if for each two consumers  $a, b \in C$  there exists a sequence of institutions  $i_1, \dots, i_k \in I$  such that  $a \in i_1$  and  $b \in i_k$  and for every  $j \in \{1, \dots, k-1\}$  it holds that  $i_j \cap i_{j+1} \neq \emptyset$ .

### 3 The Existence Theorem

In this section we state our amended version of the theorem on the existence of equilibrium in social systems with co-ordination. The theorem is based on the existence theorem of Keiding (1985, Theorem 2) in which an assumption of convexity is not mentioned. We show that the existence theorem for equilibrium in Vind (1983, Theorem 3), which Keiding claims to extend, is incorrect. Vind's assumptions are not sufficient for his assertion. Therefore the proof of the equilibrium existence theorem for an economy with bilateral exchanges, Vind (1985, Theorem 5), is invalid. Our method of proof of the theorem on the existence of equilibrium in an economy with exchange institutions can be used to give a correct proof of Theorem 5 of Vind (1983).

**Theorem 3.1** *Let  $(X, \{\beta_a, P_a, e_a\}_{a \in A})$  be a social system with co-ordination such that:*

1.  $X$  is a non-empty, convex, compact subset of  $\mathbb{R}^l$ .
2.  $\beta : X \rightrightarrows X$  is continuous with closed, convex, non-empty values.
3.  $\forall a \in A \quad P_a : X \rightrightarrows X$  has an open graph and for each  $x \in X$  it holds that  $x \notin \text{int } \overline{P_a(x)}$ . Furthermore  $\overline{P_a}$  has convex (possibly empty) values.
4.  $\forall a \in A \quad e_a : X \times X \rightarrow \mathbb{R}^l$  is continuous, and for each  $x \in X$ ,  $e_a(x, y)$  is affine in  $y$  and  $e_a(x, x) = x$ .
5.  $\forall a \in A, \forall x, y \in X \quad [e_a(x, y) \in \beta_a(x) \Rightarrow y \in \beta_a(x)]$ .
6.  $\forall x \in X \quad \beta(x) \subset \text{int } X$ .

*Then there exists an equilibrium in this social system with co-ordination.*

Instead of Assumption 3 the existence theorem of Vind (1983, Theorem 3) assumes  $P_a^c$ , which has the complements of the values of  $P_a$  as its values, and  $\overline{P_a}$ , which has the closures of the values of  $P_a$  as its values, to be continuous correspondences. Furthermore  $\overline{P_a}$  is assumed to have convex values and for every  $x \in X$  the set  $P_a(x)$  is assumed to be open in  $X$ . Finally, it is assumed that for every  $x \in X$  it holds that  $x \in \overline{P_a(x)}$ . This existence theorem of Vind (1983) is not correct, a counterexample is given by  $\Gamma := (X, (\beta_a, P_a, e_a)_{a \in A})$  with  $A := \{1\}, X := [-1, 2], \beta_1 := [0, 1], P_1(x) := X \setminus \{x\}, e_1(x, y) := y$ .

It seems difficult to give an example which shows Assumption 6 of the theorem to be necessary. The example to illustrate the necessity of the assumption in Keiding (1985) has several errors. Firstly Assumption 5 of the theorem does not hold for the example. Secondly the example has a continuum of equilibria instead of none. Attempts by the author to construct an example which shows Assumption 6 to be necessary failed.

The proof of this existence theorem is essentially the proof of Keiding (1985). In the existence theorem of Keiding the convexvaluedness of  $\beta$  was not required. The structure of the proof is a well known structure in proofs of existence of equilibria. First a correspondence is defined which satisfies the conditions of a fixed point theorem. In this case the fixed point theorem will be that of Eilenberg and Montgomery. Then the existence of a fixed point is proved. Finally it is proved that the fixed point of the correspondence is an equilibrium and therefore an equilibrium exists.

### Proof of Theorem 3.1

*Step 1. Definition and properties of  $\phi_0$ .*

Let  $A := \{1, \dots, n\}$ . Define

$$S := \{p \in \mathbb{R}^l \mid \|p\| \leq 1\},$$

where  $\|\cdot\|$  denotes the Euclidean norm. Define  $f : X \times X \times S^n \rightarrow \mathbb{R}$  such that

$$f(x, y, p_1, \dots, p_n) = \sum_{a=1}^n p_a \cdot e_a(x, y).$$

Because of Assumption 4 the function  $f$  is continuous. Furthermore by Assumption 4 it holds that  $f(x, \cdot, p_1, \dots, p_n)$  is affine.

Define  $\phi_0 : X \times S^n \rightrightarrows X$ ,

$$\begin{aligned} \phi_0(x, p_1, \dots, p_n) = \{ & y \in \beta(x) \mid f(x, y, p_1, \dots, p_n) \geq \\ & f(x, \tilde{y}, p_1, \dots, p_n) \text{ for all } \tilde{y} \in \beta(x) \}. \end{aligned}$$

By the maximum theorem [see e.g. Hildenbrand (1974, p. 29)]  $\phi_0$  is upper hemi-continuous for every  $x \in X$  and  $p \in S^n$ . Since  $\beta(x) \subset X$  is bounded by Assumption 1  $\phi_0$  is compact valued.

Furthermore  $\phi_0$  has convex values since  $f(x, \cdot, p_1, \dots, p_n)$  is affine and  $\beta(x)$  is convex by Assumption 2. Also, since  $\forall x \in X : \beta(x) \neq \emptyset$ , it follows that

$$\forall x \in X, \forall p \in S^n : \phi_0(x, p_1, \dots, p_n) \neq \emptyset.$$

*Step 2. Definition and properties of  $\phi_a$ .*

*(a) Definition of  $\phi_a$ , and a proof that  $\phi_a$  has nonempty values.*

For  $a \in \{1, \dots, n\}$  define the correspondence  $\phi_a : X \times S^n \rightrightarrows S$  by:

$$\phi_a(x, p_1, \dots, p_n) := \begin{cases} \{\hat{p}_a \in S \mid |\hat{p}_a| = 1, \forall \hat{x} \in P_a(x) \hat{p}_a \cdot \hat{x} \geq \hat{p}_a \cdot x\} & \text{if } P_a(x) \neq \emptyset. \\ S & \text{if } P_a(x) = \emptyset. \end{cases} \quad (1)$$

Since  $x \notin P_a(x)$  by Assumption 3 then, if  $P_a(x) \neq \emptyset$ , by with the separation theorem for convex sets,

$$\exists \hat{p}_a \in S : [|\hat{p}_a| = 1 \text{ and } \forall \hat{x} \in \overline{P_a(x)} : \hat{p}_a \cdot \hat{x} \geq \hat{p}_a \cdot x] \quad (2)$$

and therefore

$$\exists \hat{p}_a \in S : [|\hat{p}_a| = 1 \text{ and } \forall \hat{x} \in P_a(x) : \hat{p}_a \cdot \hat{x} \geq \hat{p}_a \cdot x]$$

so all the values of  $\phi_a$  are nonempty.

*(b) A proof that  $\phi_a$  is upper hemi-continuous and compact valued.*

For  $\phi_a(x, p_1, \dots, p_n) \subset S$  it holds that  $\phi_a(x, p_1, \dots, p_n)$  is bounded. Next we prove that  $\phi_a$  is closed correspondence, and from this it follows that  $\phi_a$  is u.h.c., because  $S$  is compact.

By definition, correspondence,  $\Psi$ , is closed at a point,  $x^0$ , if and only if

$$[x^q \rightarrow x^0, y^q \in \Psi(x^q), y^q \rightarrow y^0] \Rightarrow [y^0 \in \Psi(x^0)]$$

If  $P_a(x) = \emptyset$ , then, trivially,  $\phi_a$  is closed in  $(x, p_a, \dots, p_n)$ .

Assume that  $P_a(x) \neq \emptyset$ .

Take any sequence

$$(x^v, p_1^v, \dots, p_n^v)_{v=1}^\infty \rightarrow (x, p_1, \dots, p_n).$$

and any  $\hat{p}_i^v \in \phi_i(x^v, p_1^v, \dots, p_n^v)$ , and any  $\hat{p}_a^0 \in S$  with  $(\hat{p}_a^v)_{v=1}^\infty \rightarrow \hat{p}_a^0$ .

Clearly,  $|\hat{p}_a^0| = 1$ . Suppose that  $\hat{p}_a^0 \notin \phi_a(x, p_1, \dots, p_n)$ . Then

$$\exists \hat{x} \in P_a(x) : \hat{p}_a^0 \cdot \hat{x} < \hat{p}_a^0 \cdot x.$$



Since  $P_a(x)$  has an open graph it follows from Assumption 3 that for  $v$  sufficiently large:

$$\hat{p}_a^v \cdot \hat{x} < \hat{p}_a^v \cdot x^v \text{ with } \hat{x} \in P_a(x^v),$$

because  $(\hat{p}_a^v)_{v=1}^\infty$  converges to  $\hat{p}_a^0$  and  $x^v \rightarrow x$ .

But this contradicts (1).

So  $\hat{p}_a^0 \in \phi_a(x, p_1, \dots, p_n)$  and  $\phi_a$  is closed at every  $(x, p_1, \dots, p_n) \in X \times S^n$ , so  $\phi_a$  is u.h.c. on  $X \times S^n$ .

(c)  $\phi_a(x, p_1, \dots, p_n)$  is contractible to a point.

We shall show that  $\phi_a(x, p_1, \dots, p_n)$  is homeomorphic to a convex set. Because of the compactness of  $\phi_a(x, p_1, \dots, p_n)$  this implies contractibility of  $\phi_a(x, p_1, \dots, p_n)$  to a point.

If  $P_a(x) = \emptyset$ , then  $\phi_a(x, p_1, \dots, p_n) = S$  which is convex.

Suppose  $P_a(x) \neq \emptyset$ . Next we prove that  $P_a(x) \cap \text{rint } X \neq \emptyset$ .

If  $P_a(x) \cap \text{rint } X = \emptyset$  then every point,  $y$ , of  $P_a(x)$  is a limit-point of  $X \setminus P_a(x)$  (since  $\overline{\text{rint } X} = X$ ). Since  $P_a(x) \neq \emptyset$ , it follows that  $X \setminus P_a(x)$  is not closed, so  $P_a(x)$  is not open, which contradicts Assumption 3.

So choose  $\bar{x} \in P_a(x) \cap \text{rint } X$  and consider the set

$$\begin{aligned} Q(x) &:= \{q \in \mathbb{R}^l \mid q \cdot \bar{x} = 1, q \cdot \bar{x} \leq q \cdot x, \text{ for all } \bar{x} \in P_a(x)\} \\ &= \{q \in \mathbb{R}^l \mid q \cdot \bar{x} = 1, q \cdot \bar{x} \leq q \cdot x, \text{ for all } \bar{x} \in \overline{P_a(x)}\}. \end{aligned}$$

This set clearly is convex.

Now the map  $h : \mathbb{R}^l \setminus \{0\} \rightarrow S$  such that

$$h(\bar{p}) := \frac{\bar{p}}{|\bar{p}|}$$

is a homeomorphism from the set  $Q(x)$ , which is convex, to  $\phi_a(x, p_1, \dots, p_n)$ , so  $\phi_a(x, p_1, \dots, p_n)$  is homeomorphic to a convex set.

*Step 3. The fixed-point-theorem of Eilenberg and Montgomery.*

Define  $\phi : X \times S^n \rightrightarrows X \times S^n$  by

$$\begin{aligned} \phi(x, p_1, \dots, p_n) &:= \{(\bar{x}, \bar{p}, \dots, \bar{p}_n) \in X \times S^n \mid \bar{x} \in \phi_a(x, p_1, \dots, p_n) \\ &\quad \text{and } \forall i \in \{1, \dots, n\} : \bar{p}_i = \phi_a(x, p_1, \dots, p_n)\}. \end{aligned}$$

Since  $\phi$  is the product of  $\phi_0$  and  $\phi_a$ ,  $a \in A$ , it is upper hemi-continuous with non-empty, compact values which are contractible to a point.

By the fixed-point-theorem of Eilenberg and Montgomery [see Border (1985, p. 73)],  $\exists (x^0, p_1, \dots, p_n) \in X \times S^n$

$$(x^0, p_1^0, \dots, p_n^0) \in \phi(x^0, p_1^0, \dots, p_n^0).$$

*Step 4. The fixed-point is an equilibrium.*

Since  $x^0 \in \phi_0(x^0, p_1^0, \dots, p_n^0)$  it holds that  $x^0 \in \beta(x^0)$  and

$$\forall y \in \beta(x^0) : f(x^0, x^0, p_1^0, \dots, p_n^0) \geq f(x^0, y, p_1^0, \dots, p_n^0). \quad (3)$$

Suppose  $x^0$  is not an equilibrium.

Then  $\exists y \in X : \exists M \subset A, M \neq \emptyset$  such that:

$$e_a(x^0, y) \in P_a(x) \cap \beta_a(x^0) \text{ if } a \in M. \quad (4)$$

and

$$e_a(x^0, y) = x^0 (\in \beta_a(x^0)) \text{ if } a \notin M. \quad (5)$$

Now by Assumption 5 it follows that  $y \in \beta(x^0)$ . Furthermore:

$$\forall a \in M : P_i(x^0) \neq \emptyset.$$

Since  $p_a^0 \in \phi_a(x^0, p_1^0, \dots, p_n^0)$  it follows that

$$\forall a \in M : p_a^0 \cdot e_a(x^0, y) \geq p_a^0 \cdot x^0$$

by (1) and (4).

By Assumption 6 of the theorem, (4) and (5), it follows that for all  $a \in M$ :

$$e_a(x^0, y) \in \text{rint } X.$$

It holds that for all  $a \in M(x^0, y)$  that  $e_a(x^0, y) \in P_a(x^0) \subset \overline{P_a(x^0)}$ . Furthermore by Assumption 3 it holds that  $\overline{P_a(x^0)}$  is convex. It follows from (2), (4) and (5) that

$$p_a^0 \cdot e_a(x^0, y) \geq p_a^0 \cdot x^0 \text{ for every } a \in A.$$

Since  $x^0 \notin P_a(x^0)$ , since  $e_a(x^0, y) \in \text{int } X$ , and since  $P_a(x^0)$  is open in  $X$  this implies that

$$p_a^0 \cdot e_a(x^0, y) > p_a^0 \cdot x^0 \quad \text{for every } a \in M(x^0, y).$$

As a consequence, since  $M(x^0, y) \neq \emptyset$ ,

$$\begin{aligned} f(x^0, y, p_1^0, \dots, p_n^0) &= \sum_{a=1}^n p_a^0 \cdot e_a(x^0, y) \\ &> \sum_{a=1}^n p_a^0 \cdot x^0 \\ &= f(x^0, x^0, p_1^0, \dots, p_n^0). \end{aligned}$$

This contradicts Formula (3) So it follows that the fixed-point  $x^0$  is an equilibrium.  
Q.E.D.

Now the theorem for existence of equilibrium in an economy with exchange institutions is proven.

**Theorem 3.2** *Let  $\mathbf{E} = (\{\succeq_c, \omega_c\}_{c \in C}, I)$  be an economy with exchange institutions such that for every consumer  $c$  his preferences  $\succeq_c$  are continuous and convex. Then an equilibrium in  $\mathbf{E}$  exists.*

### Proof

The equilibria in the social system with co-ordination  $\Gamma = (Y, (\beta_a, P_a, e_a)_{a \in C \cup I})$ , which is the economy with exchange institutions  $\mathbf{E}$ , correspond to the equilibria in the social system with co-ordination  $\tilde{\Gamma} = (Y, (\tilde{\beta}_c, P_c, e_c)_{c \in C})$  where  $\tilde{\beta}_c = \beta_c \cap_{i \ni c} \beta_i$ .

Restrict the economy with exchange institutions  $\mathbf{E}$  without loss of generalization to the set  $\tilde{Y} \subset Y$  such that

$$\tilde{Y} \supset \{y : C \times I \rightarrow \mathbb{R}^I \mid \forall c \in i \in I : -\sum_{c \in C} \omega_c \leq y(c, i) \leq \sum_{c \in C} \omega_c\},$$

and  $\tilde{Y}$  is convex and compact. Such a  $\tilde{Y}$  exists. Assumptions 1, 2, 4, 5 and 6 of Theorem 3.1. are easily checked to hold for the social system with co-ordination  $\hat{\Gamma} = (\tilde{Y}, (\tilde{\beta}_c, P_c, e_c)_{c \in C})$ . Equilibria in  $\hat{\Gamma}$  correspond to equilibria in  $\mathbf{E}$  restricted to  $\tilde{Y}$  and vice versa.

Assumption 3 remains to be checked. The convexity of the values of  $\overline{P_c}$  follows directly from the convexity of  $\succeq_c$  for the agents  $c \in C$ . The irreflexivity of  $P_c$  follows from its definition and implies  $x \notin \text{int } \overline{P_c(x)}$ . The preferences of every consumer  $c$

are such that they can be represented by a continuous utility function. Furthermore the preference relation has no “thick” indifference classes except possibly for the class of satiation points.<sup>2</sup> So for the restriction of  $P_c$  to the set of net-trades excluding the satiation points the graph of  $P_c$  is open. Furthermore, because of the continuity of the preferences the set of satiation points is closed, and, therefore, its complement in the set of net-trades is open. The correspondence  $P_c$  is, by definition, empty valued for the satiation points. But then the correspondence  $P_c$  has an open graph.

Q.E.D.

## 4 The Comparison with Some Allocation Mechanisms

In this section we compare equilibrium in an economy with co-ordination with three allocation mechanisms. We examine whether these allocation mechanisms can be supported by exchange institutions.

We restrict ourselves to individualistic preference relations that, for every consumer,  $c$ , can be represented by a continuous, quasi-concave and strictly monotonous utility function,  $u_c$ . We will denote a system of such preference relations by the corresponding utility functions  $\{u_c\}_{c \in C}$ . We define  $\bar{E} := \{u_c, \omega_c\}_{c \in C}$  to be a pure exchange economy.

In the first subsection we will show that in a connected economy with co-ordination the allocation resulting from an equilibrium net trade system is Pareto efficient and individually rational in the pure exchange economy  $\bar{E}$  consisting of the consumers of the economy with exchange institutions. Furthermore every Pareto efficient allocation that is individual rational given initial endowments  $\{\omega_c\}_{c \in C}$  can be supported by a net trade system in any connected economy with exchange institutions.

In the second subsection we look at the core and find that coalitions, in this context, can be interpreted as exchange institutions. Every core allocation can be supported by an equilibrium in the economy with exchange institutions in which

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<sup>2</sup>See Debreu (1959) sections 4.6. and 4.7.



the set of exchange institutions is the set of nonempty coalitions.

Furthermore we show that the Walrasian market is supported by an exchange institution. Every Walrasian equilibrium is supported by an equilibrium in an economy with exchange institutions with the set of consumers  $C$  as the exchange institution to represent the Walrasian market.

Finally we show that not every market can be supported by an exchange institution. There may exist equilibria in a monopolistic market that are not supported by some equilibrium in the economy with exchange institutions.

## 4.1 Pareto Efficiency

In this subsection we prove that for preferences as described above the set of equilibria in a connected economy with exchange institutions supports and only supports the set of Pareto efficient allocations that are individually rational in  $\mathbf{E}$ . This will make it easier for us to make the comparisons of the next subsection because results on the Pareto efficiency of the allocation mechanisms we compare with are well known.

**Theorem 4.1** *Let  $\mathbf{E} = (\{u_c, \omega_c\}_{c \in C}, I)$  be a connected economy with exchange institutions with utility functions as defined above. Then an allocation  $x^*$  is weakly Pareto efficient and individually rational in  $\bar{\mathbf{E}} := \{u_c, \omega_c\}_{c \in C}$  if and only if there exists a net-trade system  $y^*$  that is an equilibrium in  $\mathbf{E}$  such that  $x_c^* = \sum_{i \ni c} y^*(c, i) + \omega_c$  for all consumers  $c \in C$ .*

**Proof**

**If**

Suppose  $y^*$  is an equilibrium in  $\mathbf{E}$  and  $x_c^* = \sum_{i \ni c} y^*(c, i) + \omega_c$ .

$x^*$  is individual rational for every consumer  $c$  because  $y^*$  is an equilibrium and the definition of  $\beta_c$ .

Suppose  $x^*$  is not Pareto efficient. Let  $\tilde{x}$  be individually rational for every consumer  $c \in C$  and let  $\tilde{x}$  be Pareto preferred to  $x^*$ . Define

$$M_C = \{c \in C \mid u_c(\tilde{x}_c) > u_c(x_c^*)\}.$$

$$M_I = \{i \in I \mid i \cap M_C \neq \emptyset\}.$$

$$j = \bigcup_{i \in M_I} i.$$

Connectedness of  $\mathbf{E}$  implies that every two consumers  $a$  and  $b$  from the set  $j$  can exchange every commodity through the exchange institutions of  $M_I$ .

By the continuity, the quasi-concavity and the strict monotonicity of the utility functions of the consumers of  $\mathbf{E}$  it follows, using the mean value theorem, that there exists a allocation  $\bar{x}$  such that:

1.  $\sum_{c \in M_C} \bar{x}_c < \sum_{c \in M_C} \tilde{x}_c$ .
2. for every  $c \in M_C$  it holds that  $u_c(\bar{x}_c) > u_c(x_c^*)$ .

If  $(j \setminus M_C) = \emptyset$ , then define

$$\bar{x}_c^* := \begin{cases} \bar{x}_c & \text{if } c \in M_C. \\ x_c^* & \text{if } c \notin M_C. \end{cases}$$

If  $(j \setminus M_C) \neq \emptyset$ , then define

$$t := \sum_{c \in M_C} (\tilde{x}_c - \bar{x}_c) > 0$$

and

$$\bar{x}_c^* := \begin{cases} \bar{x}_c & \text{if } c \in M_C. \\ x_c^* & \text{if } c \in C \setminus j. \\ x_c^* + \frac{1}{m}t & \text{if } c \in j \setminus M_C. \end{cases}$$

For every  $c \in j$  it holds that  $u_c(\bar{x}_c^*) > u_c(x_c^*) \geq u_c(\omega_c)$ .

By

$$\sum_{c \in j} (\bar{x}_c^* - x_c^*) = 0$$

and since every pair of consumers from  $j$  can exchange every commodity with each other through the exchange institutions of  $M_I$  it follows that there exists a  $\bar{y}$  such that for every  $c \in j$ :

$$\sum_{i \ni c} \bar{y}(c, i) + \omega_c = \bar{x}_c^*$$

and

$$\sum_{c \in i} \bar{y}(c, i) = 0.$$

Therefore  $\bar{y} \in \beta = \beta(y^*)$ .

For every  $c \in j$  it holds that  $e_c(y^*, \bar{y}) \neq y^*$  and that  $e_c(y^*, \bar{y}) \in P_c(y^*)$  since  $u_c(\bar{x}_c^*) > u_c(x_c^*)$ . Furthermore it holds that  $e_c(y^*, \bar{y}) \in \beta_c$  since  $\bar{x}_c^* \geq 0$ .

Now every  $i \in M_I$  it holds by definition that  $e_i(y^*, \bar{y}) \neq y^*$  and  $e_i(y^*, \bar{y}) \in P_i(y^*)$ . Since  $\sum \bar{y}(c, i) = 0$  it holds that  $\bar{y} \in \beta_i$ .

Therefore  $\bar{y}$  is such that Condition 2 for equilibrium in a social system with co-ordination does not hold in  $\Gamma$  for  $y^*$ . This contradicts the fact that  $y^*$  is an equilibrium in  $\Gamma$ .

### Only if

Let  $x^*$  be an allocation that is Pareto efficient and individual rational for every consumer.

Because of the connectedness of  $\mathbf{E}$  there exists a  $y^*$  such that

$$\sum_{i \ni c} y^*(c, i) = x_c^* - \omega_c$$

and

$$\sum_{c \in i} y^*(c, i) = 0.$$

Furthermore it holds that  $x_c^* \geq 0$  because  $x^*$  is feasible. It also holds that

$$\sum_{c \in C} x_c^* = \sum_{c \in C} \omega_c.$$

Therefore  $y^* \in \bigcup_{i \in I} \beta_i$  and  $y^* \in \bigcup_{c \in C} \beta_c$  by the definition of  $\beta_c$  and the individual rationality of  $x_c^*$ . So Condition 1 for  $y^*$  to be an equilibrium in  $\Gamma$  is satisfied.

Suppose  $y^*$  is not an equilibrium in  $\Gamma$ , i.e.

$$\exists \hat{y} \in \mathcal{P}(y^*) \cap \mathcal{B}(y^*).$$

This implies  $\hat{y} \neq y^*$ . Define

$$M_C := M(\hat{y}, y^*) \cap C.$$

$$M_I := M(\hat{y}, y^*) \cap I.$$

Now define  $\hat{x}_c := \sum_{i \ni c} \hat{y}(c, i) + \omega_c$ . For every  $c \in M_C$  it holds that  $u_c(\hat{x}_c) > u_c(x_c^*)$  and for  $c \notin M_C$  it holds that  $u_c(\hat{x}_c) = u_c(x_c^*)$ .

Furthermore it follows that  $\hat{x}$  is attainable because for every  $a \in A$  it holds that  $e_a(y^*, \hat{y}) \in \beta_a(\hat{y})$ , since  $\hat{y} \in \mathcal{B}(y^*)$ . But this implies  $\hat{x}$  is weakly Pareto preferred to  $x^*$  which gives a contradiction.

Q.E.D.

## 4.2 The Allocation Mechanisms

In this subsection we examine whether coalitions in the context of the core, the Walrasian market and a monopoly market can be supported by exchange institutions. Answering these questions is relatively easy since we can make use of the equivalence result of the foregoing subsection.

### 4.2.1 The Core

The first allocation mechanism we compare with the concept of equilibrium in an economy with exchange institutions is the core. We define the core as follows

**Definition 4.2** *The core of the economy  $\bar{\mathbf{E}}$ , denoted by  $C(\bar{\mathbf{E}})$ , is the set of  $x \in X$  with  $\sum_{c \in C} x_c = \sum_{c \in C} \omega_c$ , such that there does not exist a set  $F \subset C$  such that for some  $\tilde{x} \in X$  and for all  $c \in F$  it hold that:*

1.  $u_c(\tilde{x}_c) > u_c(x_c)$ .
2.  $\sum_{c \in F} \tilde{x}_c \leq \sum_{c \in F} \omega_c$ .

Define  $\mathbf{E}_C = (\{u_c, \omega_c\}_{c \in C}, I_C)$  with the consumers of this economy with exchange institutions as in  $\bar{\mathbf{E}}$  and with  $I := 2^C \setminus \{\emptyset\}$ . Now the following result holds.

**Property 4.3** *Let  $x^* \in C(\bar{\mathbf{E}})$ . Then there exists a  $y^* \in Y$  which is an equilibrium in  $\mathbf{E}_C = (\{u_c, \omega_c\}_{c \in C}, I_C)$  such that  $x_c^* = \sum_{i \ni c} y^*(c, i) + \omega_c$ .*

#### Proof

From the definition of the core it follows immediately that every core allocation is both Pareto efficient and individually rational. Furthermore  $\mathbf{E}_C$  is connected and therefore Theorem 4.1. can be applied.

Q.E.D.

So we find that a core allocation is supported by an equilibrium in the economy that has the coalitions as exchange institutions. Therefore coalitions can be interpreted as some special kind of exchange institutions for consumers with preferences as specified at the beginning of this section.

#### 4.2.2 The Walrasian Market

Next we consider the Walrasian market in the economy  $\bar{\mathbf{E}}$ . We denote the set of Walrasian allocations by  $W(\mathbf{E})$ . We interpreted a Walrasian equilibrium as stemming from an economy with only one market at which all commodities are traded. As corresponding economy with exchange institutions we therefore take  $\mathbf{E}_W = (\{u_c, \omega_c\}_{c \in C}, I_W)$  where  $I_W := \{C\}$ .

**Property 4.4** *Let  $x^* \in W(\bar{\mathbf{E}})$ , i.e.  $x^*$  is a Walrasian equilibrium allocation. Then there exists a  $y^* \in Y$  which is an equilibrium in  $\mathbf{E}_W = (\{u_c, \omega_c\}_{c \in C}, I_W)$  such that  $x_c^* = y^*(c, C) + \omega_c$ .*

#### Proof

The allocation  $x^*$  is Pareto efficient by the first theorem for welfare economics. Because there is voluntary exchange  $x^*$  is individual rational for every consumer. Finally  $\mathbf{E}_W$  is connected and therefore application of Theorem 4.1. completes the proof.

Q.E.D.

So the Walrasian market supported by some exchange institution, given the conditions we imposed on the preferences of the consumers in the beginning of this section.

#### 4.2.3 The Monopoly Market

Not every market form can be supported by some kind of exchange institution. To show this we consider an economy consisting of two consumers,  $a$  and  $b$  and two commodities, in which consumer  $a$  is the monopolist and consumer  $b$  the follower. The example shows us that in this economy the equilibrium does not correspond to an equilibrium in the economy with exchange institutions in which the exchange institution is a monopolistic market, viz.  $\mathbf{E}_M = (\{u_c, \omega_c\}_{c \in C}, I_M)$  with



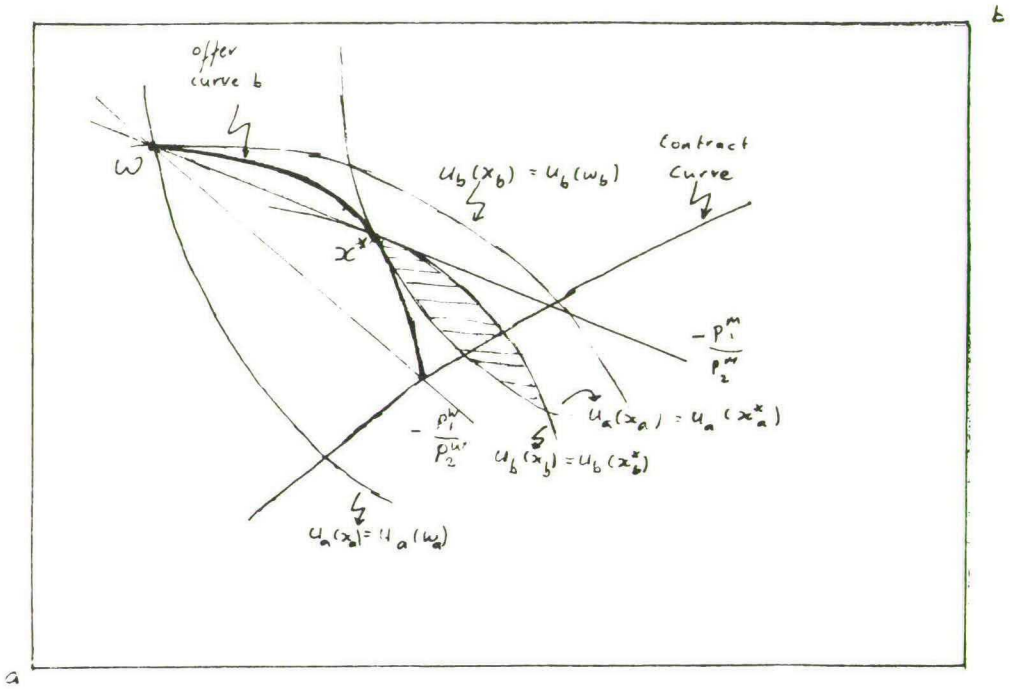


Figure 1: Monopoly pricing

$I_M := \{\{a, b\}\} = \{C\}$ . Clearly  $\mathbf{E}_M$  is connected. As the well known example depicted in the Edgeworth box in Figure 1 illustrates, the monopoly equilibrium allocation  $x^*$  need not be Pareto efficient and therefore, using Theorem 4.1., it may be the case that no equilibrium  $y^*$  in  $\mathbf{E}_M$  exists such that  $x_c^* = y^*(c, \{a, b\}) + \omega_c$ . This shows that not all markets can be supported by exchange institutions. As noted in Vind (1990) a monopolistic market can be represented by a social system with co-ordination in which the monopolist has the net-demand function of his follower in his  $e$ -function. This, however, cannot be supported by an exchange institution.

## 5 Conclusions

The aim of this comment was twofold. Firstly we wanted to show that the existence theorems for equilibrium in social systems with co-ordination of Vind (1983) and Keiding (1985) have to be amended. Secondly we wanted to analyse to what extent exchange institutions support a number of allocation mechanisms.

In Section 2 an economy with exchange institutions has been defined as a special type of social system with co-ordination in which we distinguish between consumers and exchange institutions. The concept of voluntary participation in an economy is introduced.

In Section 3 we have stated our amended version of the existence theorem for equilibrium in a social system with co-ordination. We have used this theorem to prove the existence of equilibrium in an economy with exchange institutions.

In Section 4 we assumed preferences to be individualistic and strictly monotonous. We stated that under this additional assumption that equilibria in a connected economy with exchange institutions support and only support allocations which are both individually rational and Pareto efficient. We used this equivalence result to compare equilibrium in an economy with exchange institutions with the allocation mechanisms of the core, Walrasian equilibrium and monopoly. We found that the monopolistic market is not supported by an exchange institution.

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